

Dividing & Representing RemaindersDoes the expression below show a way to represent the quotient of **586 ÷ 25** ?

Circle your choice:	Explain why you chose Yes or No:
1) $23 \text{ r } 11$ Is this a way to represent the quotient 586 ÷ 25 ? Circle One: Yes No	
2) $23\frac{11}{23}$ Is this a way to represent the quotient 586 ÷ 25 ? Circle One: Yes No	
3) 23.11 Is this a way to represent the quotient 586 ÷ 25 ? Circle One: Yes No	
4) $23\frac{44}{100}$ Is this a way to represent the quotient 586 ÷ 25 ? Circle One: Yes No	

Dividing & Representing Remainders

5) Diana wants to make gift bows. She needs 4 feet of ribbon to make one bow. With 118 feet of ribbon, how many bows can she make?

Which number of bows best represents this situation?

Circle One:

- A. $29\frac{1}{2}$
- B. 30
- C. 29
- D. $29\text{ r }4$
- E. 29.4

Explain your choice:

6) Mrs. Philbrick is packing 400 light bulbs into cartons. Each carton holds 12 light bulbs. How many cartons will she need?

Which number of cartons best represents this situation?

Circle One:

- A. 33
- B. 34
- C. 33.3
- D. $33\frac{1}{3}$
- E. 40

Explain your choice:



Dividing & Representing Remainders

This resource guides you in using the ACT cycle to implement this probe with your students and use the findings to plan instructional next steps.

Here are two examples from this 6-item probe:

Circle your choice:	Explain why you chose Yes or No:
1) $23 \text{ r } 11$ Is this a way to represent the quotient $586 \div 25$? Circle One: Yes No	

5) Diana wants to make gift bows. She needs 4 feet of ribbon to make one bow. With 118 feet of ribbon, how many bows can she make? Which number of bows best represents this situation?	
Circle One: A. $29 \frac{1}{2}$ B. 30 C. 29 D. $29 \text{ r } 4$ E. 29.4	Explain your choice:



Analyze the Assessment

What is the math?

This probe gathers information about the extent to which students can divide a three-digit number by a two-digit number and represent the remainder in a variety of formats. The last two items of the probe call upon students to determine about how to express a remainder based on the context of a word problem.

Do Students...

- Perform division of a 3-digit number by a 2-digit number and determine the remainder?

OR

- Show a limited understanding or confusion about dividing a 3-digit number by a 2-digit divisor?

Do Students...

- | | | |
|---|--|--|
| <ul style="list-style-type: none">▪ Understand different ways to express a remainder?▪ Determine how to express and interpret a remainder quotient based on the context of a real-world situation? | | <ul style="list-style-type: none">▪ Have difficulty expressing a remainder in division?▪ Misinterpret remainders when expressing answers in a real-world context? |
|---|--|--|

Oklahoma Academic Standards for Mathematics

Below is the associated standard(s) related to the intended content of this probe. This may mean a direct relationship (the content directly addresses the standard), but the content focus may instead be foundational for the standard—that is, the target may be necessary before the standard can be addressed. In this probe, students are asked to interpret representations of an expression rather than generate the representations themselves.

5.N.1.2 Divide multi-digit numbers, by one- and two-digit divisors, using efficient and generalizable procedures, based on knowledge of place value, including standard algorithms.

5.N.1.3 Recognize that quotients can be represented in a variety of ways, including a whole number with a remainder, a fraction or mixed number, or a decimal and consider the context in which a problem is situated to select and interpret the most useful form of the quotient for the solution.



Consider Students' Thinking

Examine their work

Each probe item requires a two-part response from the student: a selected response and a written explanation using words and/or pictures. Together, these two pieces of a student's answer provide important information about the student's understanding and thinking. Four possible combinations of responses are described below.

- correct selected response choice AND an explanation that provides sound reasoning
- correct selected response choice AND an explanation containing flawed or no reasoning
- incorrect selected response choice AND an explanation with reasoning that reflects some understanding
- incorrect selected response choice AND an explanation containing flawed or no reasoning

In preparation for examining your own student work, review the following:

1. the correct selected response answers;
2. student work samples showing correct selected response choices supported by sound reasoning and/or successful strategies; and
3. student work samples to illustrate common misconceptions.

1. Correct selected response choices

- | | | |
|--------|-------|-------|
| 1) Yes | 2) No | 3) No |
| 4) Yes | 5) C | 6) B |

2. Examples of correct selected response choices with sound reasoning and/or successful strategies

Students understand that one way to represent division is using a remainder notation; some students might note that this method is not as precise as using decimal or fraction notation.

<p>1) 23 r 11</p> <p><input checked="" type="radio"/> YES <input type="radio"/> NO</p>	<p>Yes because if you divide that's the answer you get.</p> $\begin{array}{r} 23 \overline{) 586} \\ \underline{50} \\ 86 \\ \underline{75} \\ 11 \end{array}$
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<p>1) 23 r 11</p> <p><input checked="" type="radio"/> YES <input type="radio"/> NO</p>	$\begin{array}{r} 25 \\ \times 23 \\ \hline 75 \\ 500 \\ \hline 575 \end{array}$ <p>$575 + 11 = 586$</p>
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<p>1) 23 r 11</p> <p><input checked="" type="radio"/> YES <input type="radio"/> NO</p>	<p>One form of the answer is 23 groups of 25 with 11 left over.</p>
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Circle Yes or No		Then, explain your thinking:	
<p>1) 23 r 11</p> <p><input checked="" type="radio"/> YES <input type="radio"/> NO</p>	<p>You don't have to make it in to fraction</p>	$\begin{array}{r} 23 \overline{) 586} \\ \underline{250} \\ 86 \\ \underline{75} \\ 11 \end{array}$	$23 \frac{11}{25}$

Examples of correct selected response choices with sound reasoning and/or successful strategies

Students understand that when expressing a remainder as a fraction, the remainder is the numerator and the divisor is the denominator.

<p>2)</p> $23 \frac{11}{23}$ <p>YES <input type="radio"/> NO <input checked="" type="radio"/></p>	<p>No because the quotient can't be the denominator of an answer which means that the divisor 25 would be the denominator $23 \frac{11}{25}$</p>
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[No because the quotient can't be the denominator of an answer which means that the divisor 25 would be the denominator $23 \frac{11}{25}$]

<p>2)</p> $23 \frac{11}{23}$ <p>YES <input type="radio"/> NO <input checked="" type="radio"/></p>	<p>no, the denominator would be 25 not 23</p>
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Students understand the relationship and distinction between the remainder, its fractional representation and its decimal representation.

<p>3)</p> 23.11 <p>YES <input type="radio"/> NO <input checked="" type="radio"/></p>	<p>I circled no because .11 is the same as $\frac{11}{100}$ and the fraction that is shown above; the denominator is 25.</p>
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<p>3)</p> 23.11 <p>YES <input type="radio"/> NO <input checked="" type="radio"/></p>	<p>No because remainder 11 is not a decimal.</p>
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Examples of correct selected response choices with sound reasoning and/or successful strategies

<p>Student understands that equivalent fractions can be used to express the quotient.</p>	<p>4) $23 \frac{44}{100}$</p> <p><input checked="" type="radio"/> YES NO</p> <p>I circled yes because the answer that I got from dividing 586 by 25 was $\frac{1}{25}$. To know that $\frac{1}{25}$ is the same as $\frac{44}{100}$ if you multiply it by 4.</p>
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<p>Student interprets a remainder in context in order to determine the appropriate way to round when expressing the answer.</p>	<p>6) Mrs. Philbrick is packing 400 light bulbs into cartons. Each carton holds 12 light bulbs. How many cartons will she need? Which number of cartons best represents this situation?</p> <p>Circle One:</p> <p>A. 33 <input checked="" type="radio"/> B. 34 C. 33.3 D. 33 $\frac{1}{3}$ E. 40</p> <p>Explain your choice:</p> <p>She will be able to fill 33 cartons with 12 light bulbs in each, but she will have four extra bulbs which will need an extra carton.</p>
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3. Examples that reflect common misconceptions

<p>Does not distinguish whole number remainders from decimal and fraction representations</p> <p>Students interpret the decimal point and "r" as meaning the same thing.</p>	<p>3) 23.11</p> <p><input checked="" type="radio"/> YES NO</p> <p>because that means 23 r 11,</p>
	<p>3) 23.11</p> <p><input checked="" type="radio"/> YES NO</p> <p>because the part is after the twenty three there for making it be a remainder</p>

Examples that reflect common misconceptions

Expects that all representations must contain the same numbers

Students do not consider remainders expressed as equivalent fractions.

4) $23 \frac{44}{100}$

No because it isn't 44 and where did the 100 come from?

4) $23 \frac{44}{100}$

Yes No

those aren't the numbers in the problem.

Fraction misconceptions

Students overgeneralize ideas about fractions or misapply algorithms

2) $23 \frac{11}{23}$

YES NO

Yes because in a fraction the smaller number goes on top and $\frac{11}{23}$ 11 is smaller than 23.

4) $23 \frac{44}{100}$

Yes No

because when I multiply $23 \times 100 + 44$ I get 5400.

Uses a fraction representation rather than rounding

Student does not consider the meaning of the fractional part in the context of the problem.

6) Mrs. Philbrick is packing 400 light bulbs into cartons. Each carton holds 12 light bulbs. How many cartons will she need?
Which number of cartons best represents this situation?

Circle One:

- A. 33
- B. 34
- C. 33.3
- D. 33 $\frac{1}{3}$
- E. 40

Explain your choice:

I got 33 and $\frac{1}{3}$ because when I divided 400 by 12 I got 33 and $\frac{4}{12}$. I know that $\frac{4}{12}$ simplifies to $\frac{1}{3}$ so my answer is 33 and $\frac{1}{3}$



Take Action

Move student learning forward

Instructional ideas to consider

- Build understanding of division language such as “has how many.” For example, $586 \div 25$ can be thought of as “586 has how many groups of 25?” Give students opportunities to reason about the quotient and whether 586 can be divided into groups of 25 evenly with nothing leftover or whether it has a remainder.
- Give students practice with different methods of division, including partial quotients, short division and the standard long division algorithm; help them see connections between the methods. Discuss how inverse operations can be employed to check answers and test equivalence.
- Help students build understanding of the meaning of a remainder as part of a group that can be described as a fraction with the numerator being the quantity left over and the dividend being the divisor or the quantity you are dividing by.
- Use visual models such as hundreds grids to build understanding of equivalent fractions and to express fractions in decimal form. Interactivate [www.shodor.org/interactivate/] has lessons and activities such as Fraction Finder Fraction Pointer, and Fraction Quiz that allow the user to visually experiment with and compare the values of fractions and decimals using areas models and the number line. <http://www.shodor.org/interactivate/activities/>
- Provide opportunities for students to think about division problems in the context of a real world situation. Discuss what the dividend, divisor and quotient mean in the context of the problem. Also, discuss what representation of the answer makes the most sense – representing with a remainder expressed as a fraction or decimal, rounding down or rounding up to the next whole number. To reinforce key ideas, give students opportunities to come up with their own real world contexts and write and share word problems.
- As always, consider which of the Mathematics Actions and Processes will be the focus of your instruction. (i.e. have students defend their choices to other students to support ability to communicate using mathematical reasoning)

Sample Hinge-point Question to Assess Progress

Here is one example. You will likely need to create additional hinge-point questions as you implement targeted instruction to address learning needs.

48 students are sharing 84 cupcakes. How could you describe how many cupcakes they each have? Chose all that apply.

- A. 2 cupcakes
- B. 1.75 cupcakes
- C. 1 r 36 cupcakes
- D. $1\frac{3}{4}$ cupcakes
- E. 1 cupcake



Attributed to the work of Rose Tobey, Arline, Fagan.
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