Ways to Show 4 x 3
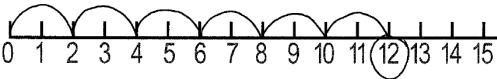
Decide if the example shows a way to think about   **4 x 3**

Circle your choice:	Explain why you chose Yes or No:
A) 4 + 4 + 4 YES NO	
B) YES NO	
C) 3, 6, 9, 12 YES NO	

Ways to Show 4 x 3

Decide if the example shows a way to think about **4 x 3**

Circle your choice:	Explain why you chose Yes or No:
D) 0 1 2 3 4 5 6 7 8 9 10 11 12 YES NO	
E) ● ● ● ● ● ● ● ● ● ● ● ● YES NO	
F) YES NO	

Ways to Show 4 x 3

This resource guides you in using the ACT cycle to implement this probe with your students and use the findings to plan instructional next steps.

Here is one example from this 6-item multiplication probe:

Decide if the example shows a way to think about 4 x 3

Circle Yes or No	**Then, explain your thinking:**
E) • • • • • • • • • • YES NO	

Analyze the Assessment

What is the math?

This probe gathers information about the extent to which students can interpret a variety of ways to represent a multiplication expression, assessing understanding of important multiplication ideas including "groups of," repeated addition, skip counting, and visual representations such as the number line, an array or an area model.

Do Students...		
▪ Understand that multiplication can be represented in a variety of ways numerically and visually (repeated addition, skip counting, arrays, area models and number line)? ▪ Consider the magnitude of the factors when evaluating the representations?	**OR**	▪ Show a limited understanding that is more focused on numeric multiplication representation rather than visual representation? ▪ Focus on the product (12) and select any representations based on that alone?

Oklahoma Academic Standards for Mathematics

Below is the associated standard(s) related to the intended content of this probe. This may mean a direct relationship (the content directly addresses the standard), but the content focus may instead be foundational for the standard—that is, the target may be necessary before the standard can be addressed. In this probe, students are asked to interpret representations of an expression rather than generate the representations themselves.

> **3.N.2.1** Represent multiplication facts by using a variety of approaches, such as repeated addition, equal-sized groups, area models, equal jumps on a number line and skip counting.

Consider Students' Thinking → Examine their work

Each probe item requires a two-part response from the student: a selected response and a written explanation using words and/or pictures. Together, these two pieces of a student's answer provide important information about the student's understanding and thinking. Four possible combinations of responses are described below.
- correct selected response choice AND an explanation that provides sound reasoning
- correct selected response choice AND an explanation containing flawed or no reasoning
- incorrect selected response choice AND an explanation with reasoning that reflects some understanding
- incorrect selected response choice AND an explanation containing flawed or no reasoning

In preparation for examining your own student work, review the following:
1. the correct selected response answers;
2. student work samples showing correct selected response choices supported by sound reasoning and/or successful strategies; and
3. student work samples to illustrate common misconceptions.

1. Correct selected response choices

A) YES	D) YES
B) NO	E) NO
C) YES	F) YES

Multiplication can be thought of as repeated addition (Choice A: 4 + 4 + 4) and also as an area model with 4 rows and 3 columns or 3 columns and 4 rows spanning an area of 12 units (Choice F). Skip counting by threes (3, 6, 9, 12) is a way of thinking about 4 groups of 3 or repeatedly adding 4 of the 3's (Choices C and D).

2. Examples of correct selected response choices with sound reasoning and/or successful strategies

Student understands that repeated addition is one way to think of multiplication. 4 x 3 can be interpreted as 4, 3 times or 4 + 4 + 4.	A) 4 + 4 + 4 (YES) NO	Yes because four three times equals 12,

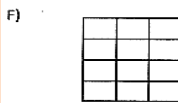|		
Student understands that skip counting, repeated adding and multiplication are related.	C) 4, 8, 12 (YES) NO	It's just like doing multiplication it's you a different way which is counting by fours.

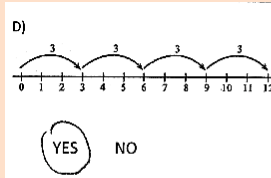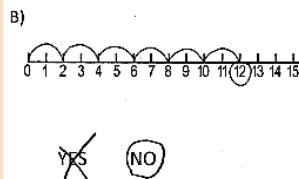|		
Student understands that multiplication can be shown visually in grid or dot arrays in which the factors are the dimensions.	F) [grid] (YES) NO	It's a grid of 3x4.
	E) [dot array] YES (NO)	It's in the form of 2x6. It still has the same answer though
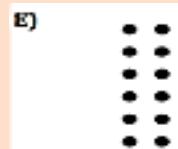
Examples of correct selected response choices with sound reasoning and/or successful strategies

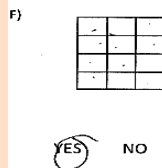|	
Student understands that jumps of an equal length on a number line can represent multiplication. One factor represents the size of the jump and the other factor represents the number of jumps**.**	**D)** *(number line with 4 jumps of 3, ending at 12; YES circled, NO)* — "Because there is 4 groups of 3." **B)** *(number line, 11-12-13 14 15 region marked; YES crossed out, NO circled)* — "it equallys 12 but its counting by 2 not 3 so it does not Show 4x3."

3. Examples that reflect common misconceptions
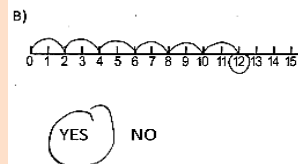
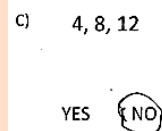|	
Multiplication is just a set of facts Students know that 4 x 3 has a value of 12 but they are not thinking about the size of the factors or "groups of" idea of multiplication.	**F)** *(grid model; YES circled, NO)* — "it counts up fo 12" This student focuses on the count shown in the model (12) . It's not clear whether the student also sees the factors of 4 and 3 in this **B)** *(number line; YES circled, NO)* — "becacuse 4 X 3 ⊏ 12 and it stop on 12." This student focuses on the total result of 12 without noting that this number line shows 6 jumps of size 2 units.
Skip counting might be a fun thing to do, but it doesn't have anything to do with multiplication. Students don't see a connection between skip counting and repeated addition or "groups of".	**C)** 4, 8, 12 YES (NO circled) — "thats not even multiplying it's counting by 4. it could be multiplying but in this case your counting by 4s."

Examples that reflect common misconceptions

Multiplication and addition are two unrelated operations Students fail to recognize that one way of thinking of multiplication is as "shorthand" for addition.	A) 4 + 4 + 4 YES ~~NO~~ (circled) *because we are not adding.* A) 4 + 4 + 4 YES (NO circled) *4x3 is the way to write it. because 4x4x4 is 4 and 3 times but it doesn't equel 4x3.*

T ake Action → Move student learning forward

Instructional ideas to consider

- Developing a deep and flexible thinking about the operation of multiplication with whole numbers will help students as they expand their experiences with multiplication to multii-digit numbers, multiplication of other rational numbers such as fractions, decimals and integers and eventually to multiplication involving variables.

- Have students work with concrete materials, arrays, area models and number lines to build understanding of multiplication as a number of groups of a particular quantity. Or, to see that the product of two factors can be represented by the area of a number of dots contained in an array.

- Use the number line representation to help reinforce multiplication as a certain number of groups of a particular quantity. On the number line this is represented by the size of a "jump" and the number of jumps.

- Give students opportunities to explore skip counting and its connection to multiplication.. The quantity they are skip counting by is the magnitude of the group and the number of times they skip count by that quantity is the number of groups.

- Provide opportunities for students to solve multiplication problems in context by giving them story problems. The context of a situation can help students create a visual representation that shows the factors and product. A context can also support help make a distinction that while 3 x 4 and 4 x 3 have the same product, they are not the same.

- Help students to see that one way, but not the only way, to think about multiplication is as repeated addition. Later, this idea of repeated addition can help them to see division as repeated subtraction of a particular quantity.

- Building understanding of the meaning of multiplication and how it can be visually represented can help students who are learning their multiplication facts. Using benchmark factors to compute multiplication is also helpful. For example, a student can compute 7 x 6 by using a related but easier fact, 7 x 5 = 35, then adding 7.

- As always, consider which of the Mathematics Actions and Processes will be the focus of your instruction. (i.e. have students defend their choices to other students to support ability to communicate using mathematical reasoning)
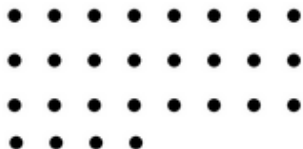

Sample Hinge-point Question to Assess Progress

Here is one example. You will likely need to create additional hinge-point questions as you implement targeted instruction to address learning needs.

Which approaches are ways to think about or represent 7 x 4? (Choose all)

     a) 7, 14, 21, 28, 35, 42

     b) 7 x 5 - 7

     c)

     d) 4 x 7

     e) 25 + 3