Name _____

Estimate Fraction Sums

Without actually calculating, use what you know about fractions to estimate the sum.

Circle the best estimate:	**Explain your choice:**
1) $\frac{1}{7}+\frac{1}{10}$ A) $\frac{1}{12}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) 2 E) 17	
2) $\frac{8}{9}+\frac{11}{12}$ A) $\frac{1}{2}$ B) 1 C) 2 D) 19 E) 21	
3) $\frac{5}{9}+\frac{3}{5}$ A) $\frac{1}{2}$ B) 1 C) 2 D) 6 E) 8	
4) $\frac{2}{5}+\frac{1}{6}$ A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) 3 E) 11	

Estimate Fraction Sums

This resource guides you in using the ACT cycle to implement this probe with your students and use the findings to plan instructional next steps.

Here is one example from this 4-item number line probe:

Without actually calculating, use what you know about fractions to estimate the sum.

Circle the best estimate:	Explain your choice:
1) $\frac{1}{7}+\frac{1}{10}$ A) $\frac{1}{12}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) 2 E) 17	

Analyze the Assessment → What is the math?

This probe gathers information about the extent to which students can (1) reason about the magnitude of a fraction (2) apply a strategy for estimating the sum of two fractions.

Do Students…		
▪ Understand the relationship between the numerator and denominator in determining the magnitude of a fraction? ▪ Use knowledge of common benchmark fractions and or visual representations to reason about the magnitude of fractions? ▪ Show an understanding of what it means to estimate a sum of two fractions?	**OR**	▪ Treat the values in the numerator or denominator as whole numbers when estimating a sum? ▪ Use flawed reasoning about the magnitude of a fraction compared to a common benchmark? ▪ Rely upon common denominator procedures to calculate rather than estimate a sum of two fractions?

Oklahoma Academic Standard for Mathematics

Below is the associated standard related to the intended content of the probe. This may mean a direct relationship (the content directly addresses the standard), but the content focus may instead be foundational for the standard—that is, the target may be necessary before the standard can be addressed.

- **5.N.3.1** Estimate sums and differences of fractions with like and unlike denominators, mixed numbers, and decimals to assess the reasonableness of the results.

Consider Students' Thinking → Examine their work

Each probe item requires a two-part response from the student: a selected response and a written explanation using words and/or pictures. Together, these two pieces of their answer provide important information about students' understanding and thinking. Four possible combinations in students' responses are described below.

- correct selected response choice AND an explanation that provides sound reasoning
- correct selected response choice AND an explanation containing flawed or no reasoning
- incorrect selected response choice AND an explanation with reasoning that reflects some understanding
- incorrect selected response choice AND an explanation containing flawed or no reasoning

In preparation for examining your own student work, review the following:

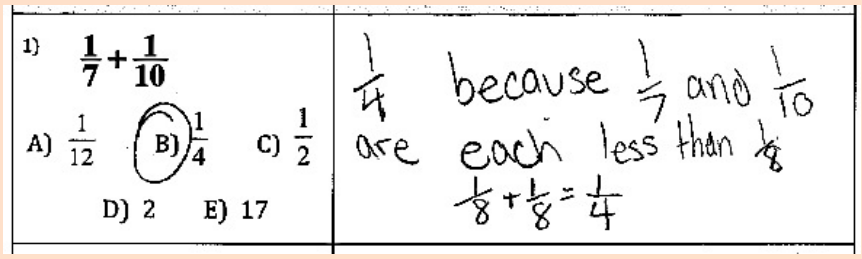
1. the correct selected response answers;
2. student work samples showing correct selected response choices supported by sound reasoning and/or successful strategies; and
3. student work samples to illustrate common misconceptions.

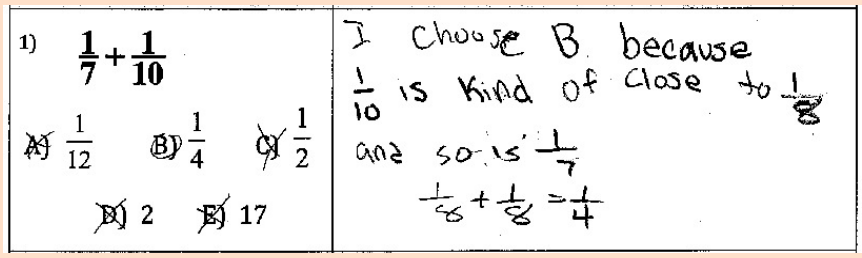
1. Correct selected response choices

1) B 2) C 3) B 4) B

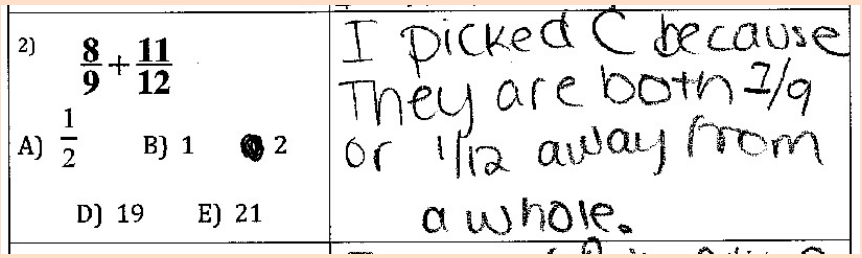
2. Examples of correct selected response choices with sound reasoning and/or successful strategies

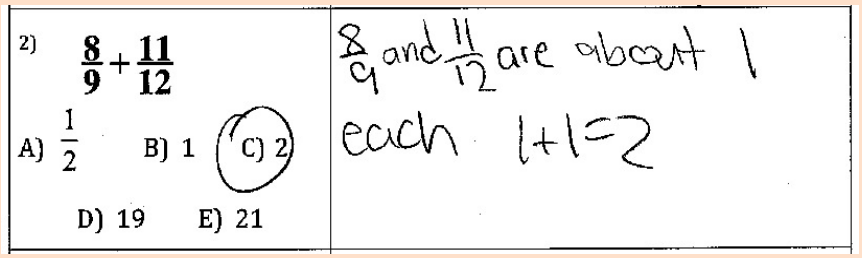
Students reason about the magnitude of unit fractions in comparison to a common benchmark unit fraction (1/4) in order to estimate the sum of two unit fractions.

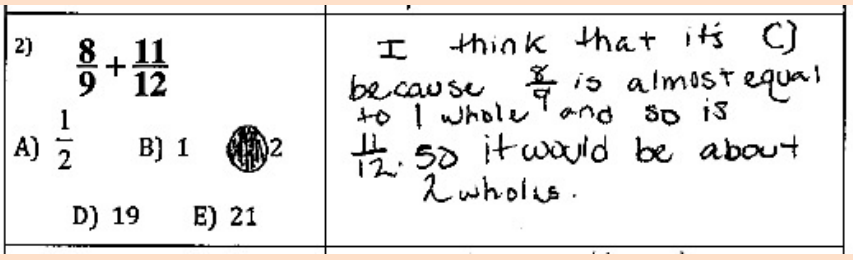




Students reason about the magnitude of each fraction in comparison to one whole to determine an estimate.

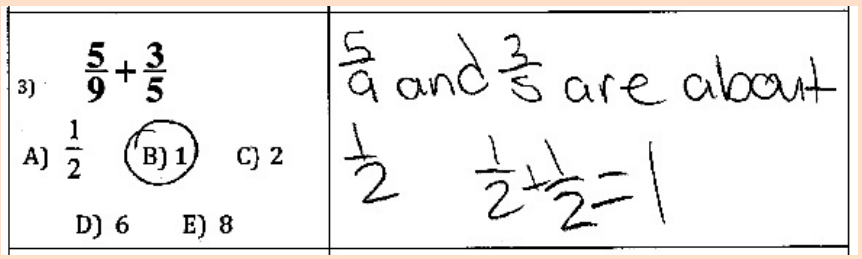


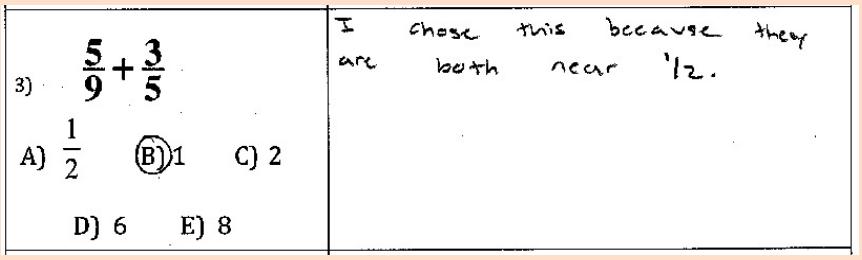




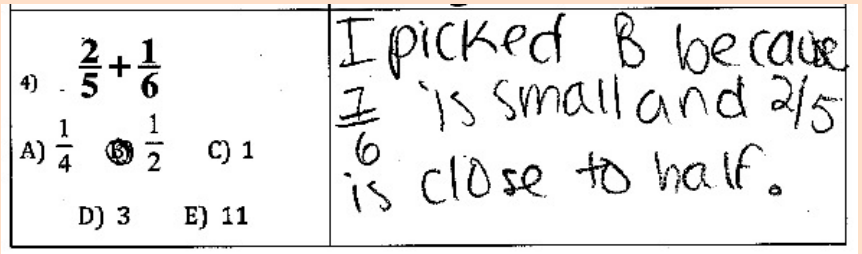
Examples of correct selected response choices with sound reasoning and/or successful strategies

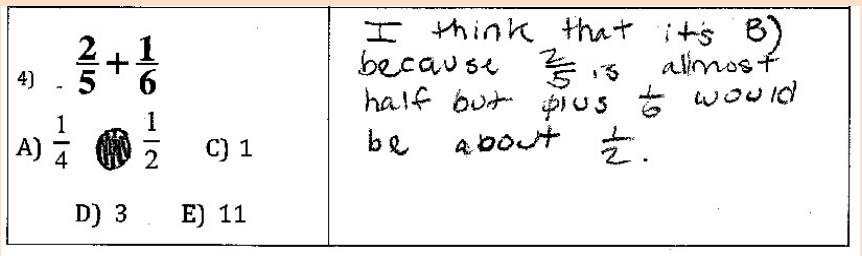
Students reason about a fraction's relationship to ½ to make an estimate of a sum.





Student reasons about unit and non-unit fractions to estimate a sum.

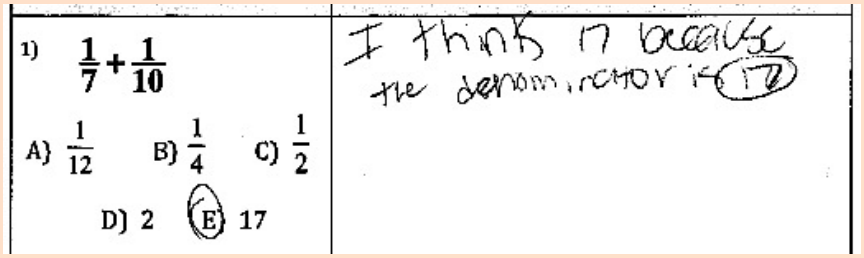


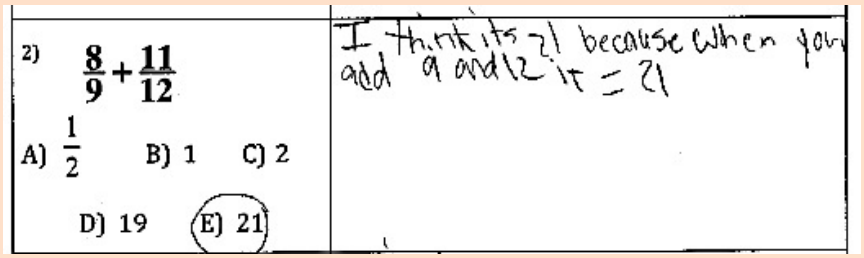


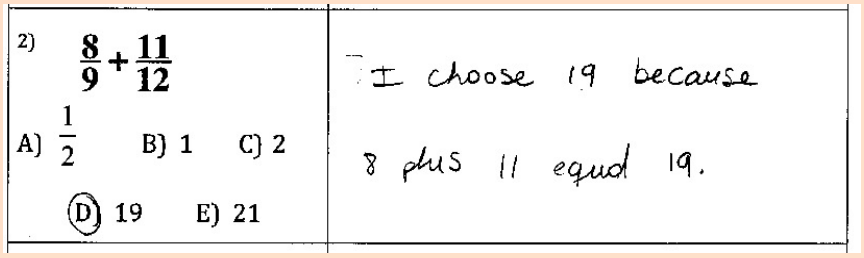
3. Examples that reflect common misconceptions

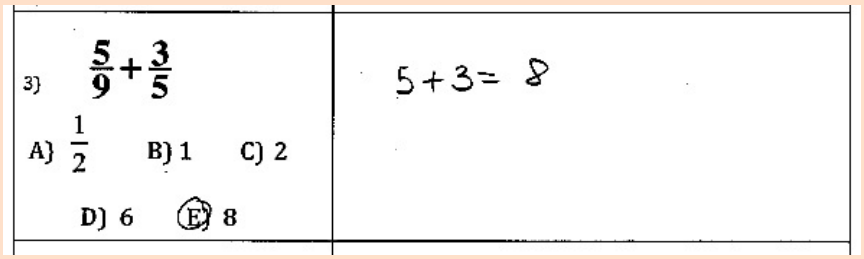
Adds numerators or denominators

Students treat fractions as though they are whole numbers and determine the sum by adding the numerators or denominators.







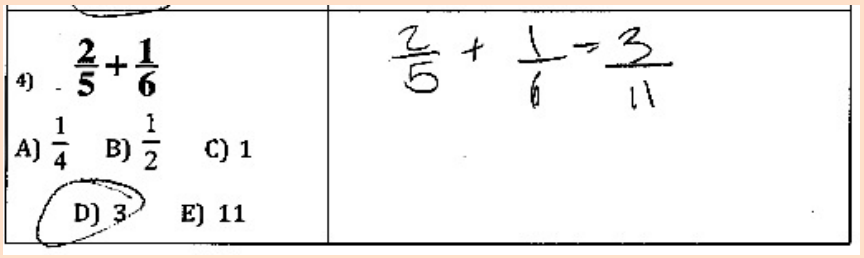


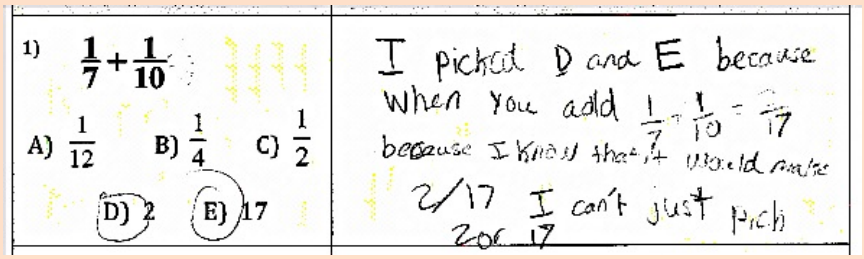
Examples that reflect common misconceptions

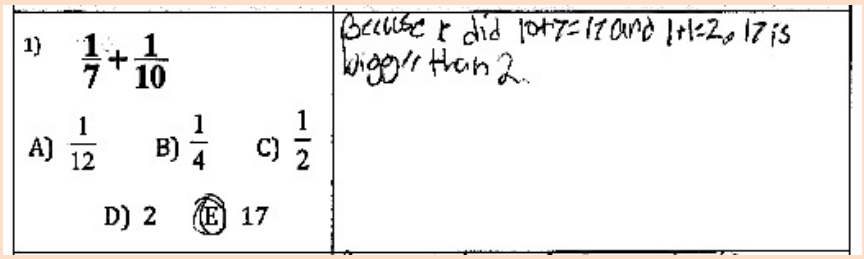
Adds both numerators and denominators

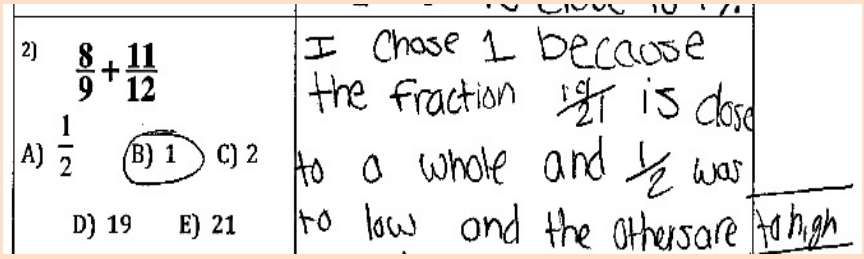
Students treat fractions as though they are whole numbers and add the numerators and denominators to determine the sum. As illustrated by these samples, students interpret the answer in a variety of ways including:

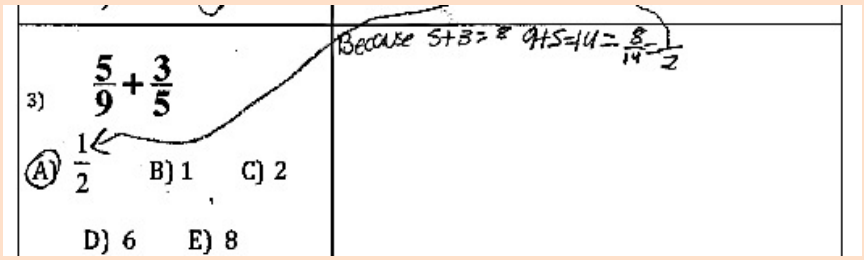
- Choosing the answer that matches the sum of the numerators or the sum of the denominators.
- Choosing two answers
- Choosing the larger sum
- Estimating the value of the fraction resulting from adding numerators and denominators
- Determining the difference between the denominator and numerator.

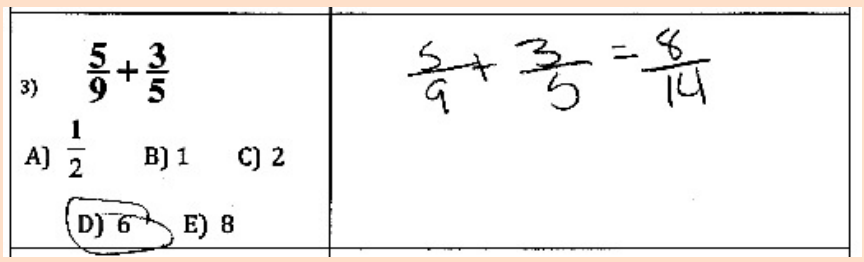








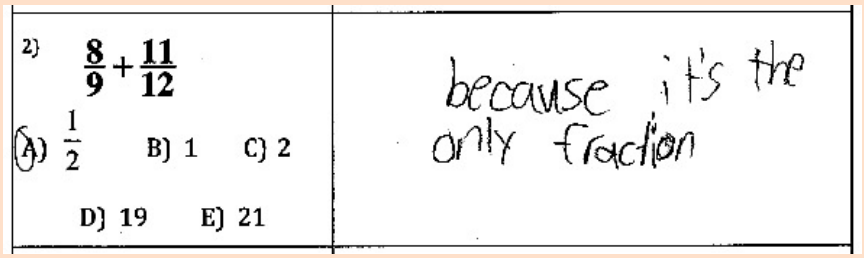


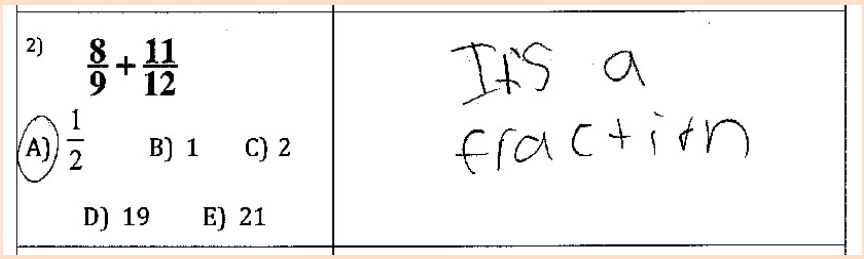


Examples that reflect common misconceptions

The sum of two fractions must also be a fraction.

Some students eliminate the whole number choices because they mistakenly believe that the estimate of the sum must be fraction, not a whole number.

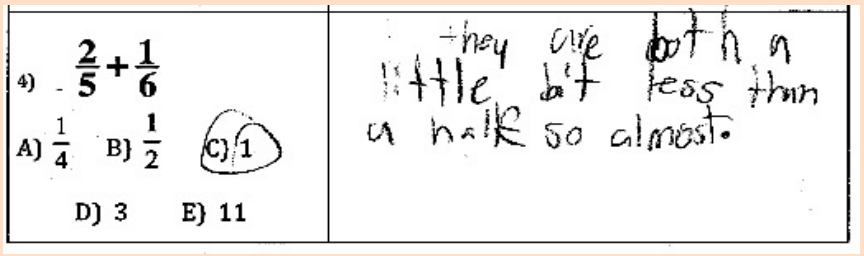


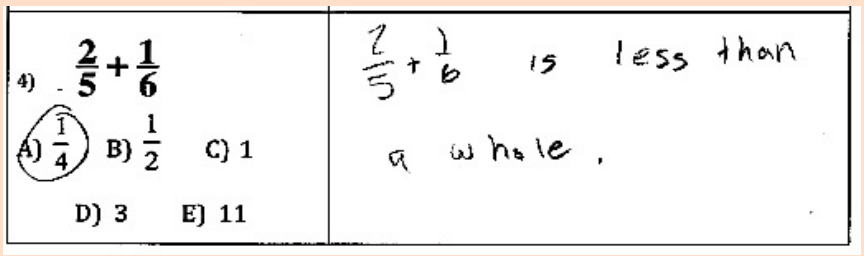


Imprecise reasoning

The first student has flawed reasoning about the magnitude of a fraction that impacts their estimate of the sum (1/6 is more than a little bit less than ½).

In the second sample, the student is correct that the sum is less than a whole but the student didn't explain how he/she eliminated ½ as the answer. Teacher would need to have a conversation to determine where the error is.

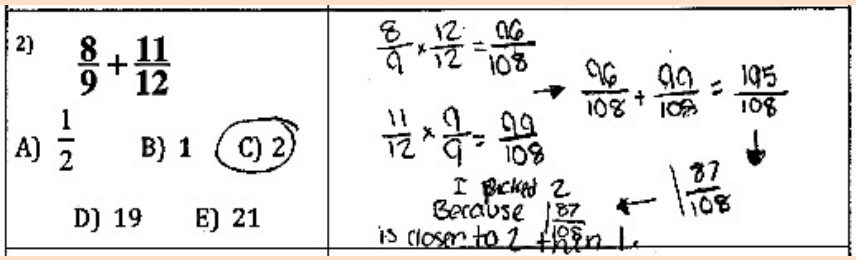


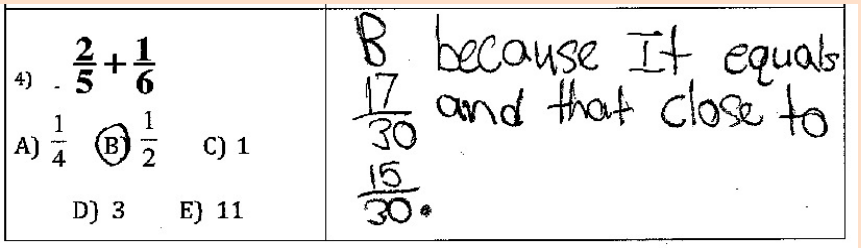


Examples that reflect common misconceptions

Computes rather than estimates

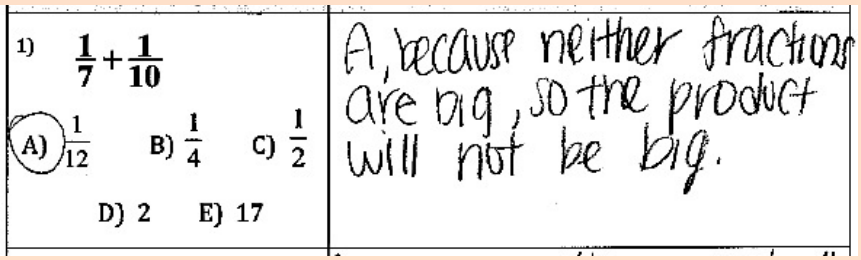
Some students will apply the common denominator algorithm to compute the sum, raising questions about their understanding of and ability to estimate.

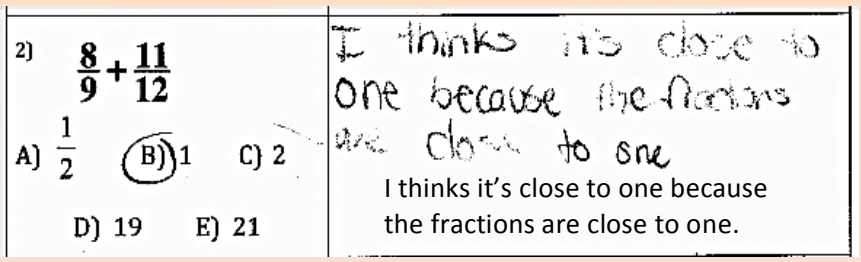




Misinterprets the meaning of the expression

Some students fail to reason about the sum of the fractions; they may reason about the magnitude of the fractions or may apply an operation other than addition.



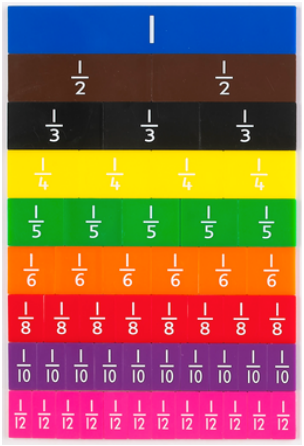


I thinks it's close to one because the fractions are close to one.

Instructional ideas to consider

- Estimating a sum of two fractions can present challenges for students because estimation depends on conceptual understanding of a fraction, being able to reason about its magnitude and understanding the effect of adding two fractions. Help students to:
  - Understand that a fraction is a number with (1) a precise magnitude that can be represented with an area model and (2) a precise location on the number line;
  - Understand how the numerator and denominator each contribute to the value (e.g. piece size and number of pieces and interval length on number line and number of jumps);
  - Compare fractions to common benchmarks such as ¼, ½ and 1; and
  - Understand addition as combining quantities. The meaning of addition remains the same as for whole numbers even when the procedures are different.

- Use a variety of concrete materials to help students:
  - build visual images of fractions;
  - understand how the numerator and denominator contribute to the fraction's magnitude;
  - compare fractions to common benchmarks such as ¼, ½ and 1.



- Use area models and the number line representation to build understanding of fraction addition, the need for a common denominator and as visual tools for estimating fraction sums.

- Help reinforce visual images of fractions with the language you use to name and describe fractions. Rather than limiting your language to "three over five" or "three out of five" to describe three fifths, use language like "three one-fifths" or "three of the one-fifth pieces." Research has shown that this language can help students move away from the misconception of treating numerators and denominators like whole numbers and adding them. This language also helps students understand the need for a common denominator. In order to add 3 one-fifths and 7 one-tenths, you will need to express the sum with a common-size piece: tenths.

- Use computer tools to build visual images of fractions and the meaning of addition.

  Fraction Track http://illuminations.nctm.org/Activity.aspx?id=4148

  Fraction Pointer http://www.shodor.org/interactivate/activities/FractionPointer/

  Fraction Finder http://www.shodor.org/interactivate/activities/FractionFinder/

  Fraction Sorter http://www.shodor.org/interactivate/activities/FractionSorter/

  Visual fractions http://www.visualfractions.com/Games.htm

- Provide opportunities for students to estimate and discuss their approaches for determining a good estimate. Build reasoning and estimation skills in combination with computational fluency and discuss how estimates and actual computations compare. Some students value precise computation over estimating but estimating can build and reveal conceptual understanding.

- Solve problems in real world contexts to help strengthen students' understanding and their ability to apply ideas. Contexts can help students select appropriate concrete materials, and strengthen their abilities to visualize and represent fractions.

- As always, consider which of the Mathematics Actions and Processes will be the focus of your instruction (i.e. have students defend their choices to other students to support ability to communicate using mathematical reasoning).

Sample Hinge-point Question to Assess Progress

Here are two examples. You will likely need to create additional hinge-point questions as you implement targeted instruction to address learning needs.

1. Which is the best estimate of $\frac{4}{5} + \frac{8}{9}$?

a. $\frac{1}{2}$ **b.** 1 **c.** 2 **d.** 12

2. Which is the best estimate of $\frac{4}{7} + \frac{2}{5}$?

a. $\frac{1}{2}$ **b.** 1 **c.** 2 **d.** 6

Correct selected response choices for Hinge-point questions.

1. c 2. b