Name ______________________________

Size of Decimals

For each problem below, determine the best answer by using what you know about the value of the fractions and decimals in the problem.

Circle your choice:	**Explain or show why you choose that fraction:**
1) **Which fraction is closest in value to 0.68?** a) $\frac{1}{70}$ b) $\frac{1}{7}$ c) $\frac{2}{3}$ d) $\frac{6}{8}$	
2) **Which fraction is closest in value to 0.47?** a) $\frac{1}{5}$ b) $\frac{1}{45}$ c) $\frac{1}{2}$ d) $\frac{4}{7}$	
3) **Which decimal is closest in value to $\frac{3}{49}$?** a) **0.16** b) **0.349** c) **0.003** d) **0.06**	
4) **Which decimal is closest in value to $\frac{4}{105}$?** a) **0.4105** b) **0.04** c) **0.004** d) **0.25**	

Comparing Numbers: Decimals and Fractions

This resource guides you in using the ACT cycle to implement this probe with your students and use the findings to plan instructional next steps.

Here is one example from this 4-item comparing numbers probe:

Circle your choice:	Explain or show why you choose that fraction:
1) **Which fraction is closest in value to 0.68?** a) $\frac{1}{70}$ b) $\frac{1}{7}$ c) $\frac{2}{3}$ d) $\frac{6}{8}$	

Analyze the Assessment — What is the math?

This probe gathers information about the extent to which students use reasoning about place value and magnitude when comparing decimals and fractions.

Do Students…		
▪ Reason about the relationship between the sizes of quantities by: ○ comparing using a common benchmark? ○ expressing in decimal form then comparing? ○ expressing in fraction form then comparing? ▪ Provide mathematical justifications to explain the relationship between the quantities (i.e. use of visual models, number lines, place value reasoning, common denominators, etc.)?	**OR**	▪ Apply incorrect reasoning about the relationship of the size of quantities by: ○ misjudging the size of the quantity as compared to the benchmark number? ○ associating the decimal point as having the same meaning as the fraction bar? ▪ Provide inaccurate mathematical justifications that show lack of conceptual understanding of relationships between numbers?

Oklahoma Academic Standards for Mathematics

Below is the associated standard related to the intended content of the probe. This may mean a direct relationship (the content directly addresses the standard), but the content focus may instead be foundational for the standard—that is, the target may be necessary before the standard can be addressed.

> **5.N.2** Read, write, represent, and compare fractions and decimals; recognize and write equivalent fractions; convert between fractions and decimals; use fractions and decimals in real-world and mathematical situations.

Consider Students' Thinking

Examine their work

Each probe item requires a two-part response from the student: a selected response and a written explanation using words and/or pictures. Together, these two parts provide important information about the student's understanding and thinking. Four possible combinations of student responses are described below.

- correct selected response choice AND an explanation that provides sound reasoning
- correct selected response choice AND an explanation containing flawed or no reasoning
- incorrect selected response choice AND an explanation with reasoning that reflects some understanding
- incorrect selected response choice AND an explanation containing flawed or no reasoning

In preparation for examining your own student work, review the following:

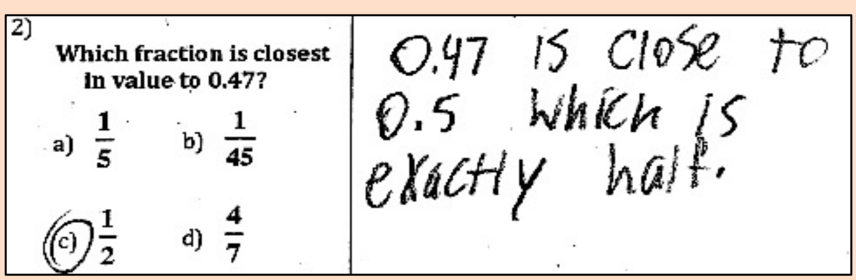
1. the correct selected response answers;
2. student work samples showing correct selected response choices supported by sound reasoning and/or successful strategies; and
3. student work samples to illustrate common misconceptions.

1. Correct selected response choices

1) c. $\frac{2}{3}$ 2) c. $\frac{1}{2}$ 3) d. 0.06 4) b. 0.04

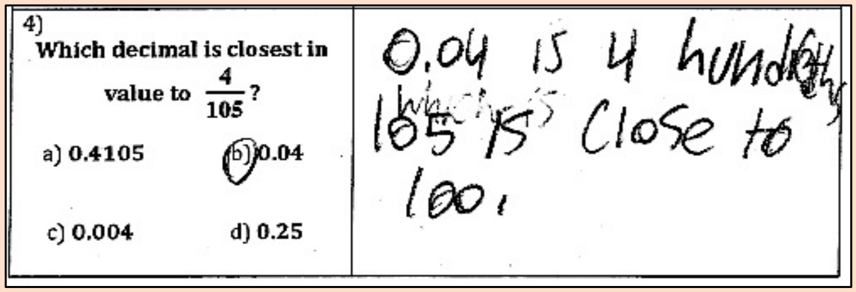
2. Examples of correct selected response choices with sound reasoning and/or successful strategies

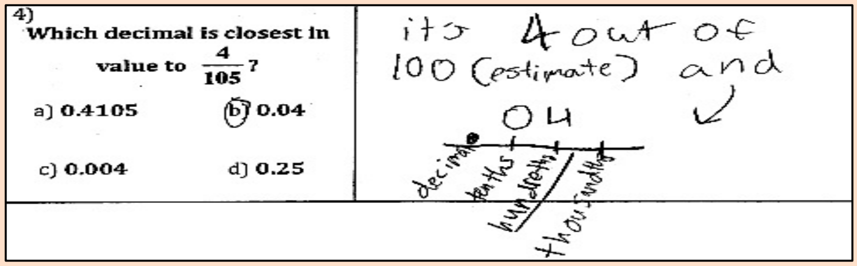
Student successfully compares the decimal and fractions by recognizing 0.47 is close to $\frac{1}{2}$.



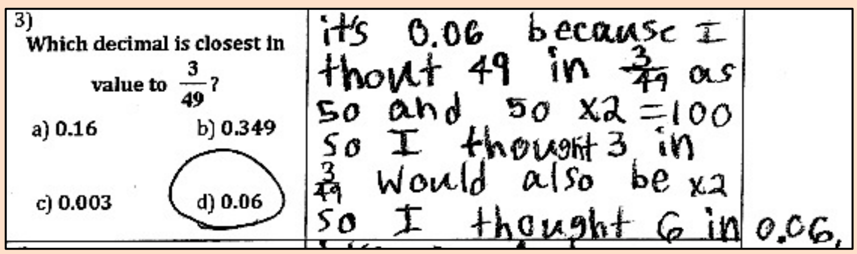
Examples of correct selected response choices with sound reasoning and/or successful strategies

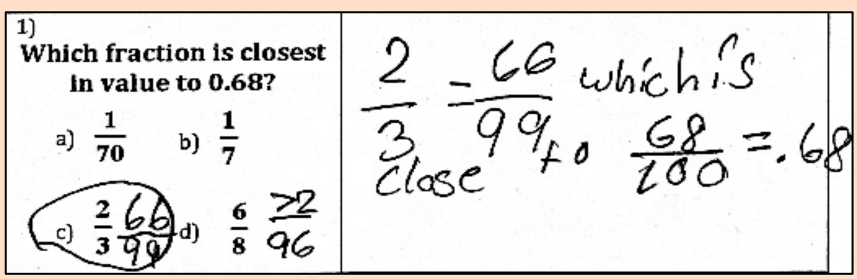
Students successfully compare the fraction and decimals by reasoning about place value.



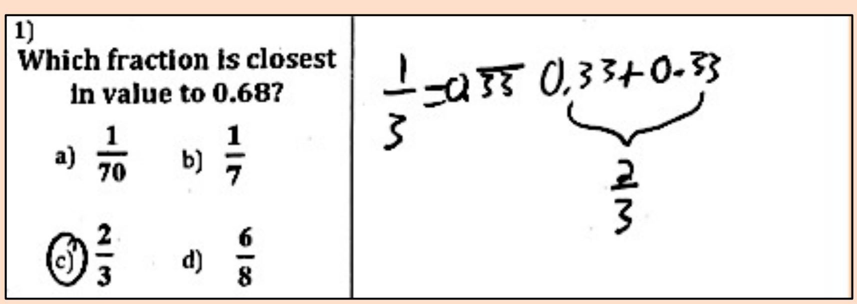


Students successfully compare fractions and decimals using equivalent fractions and reasoning about place value.





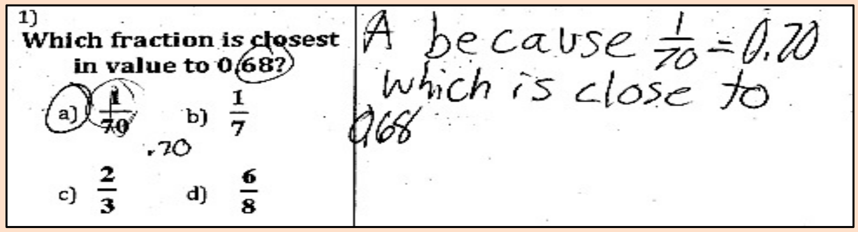
Student successfully compares the decimal and fractions and justifies choice using decimal equivalences.



3. Examples that reflect common misconceptions

Incorrect conversion to a unit fraction

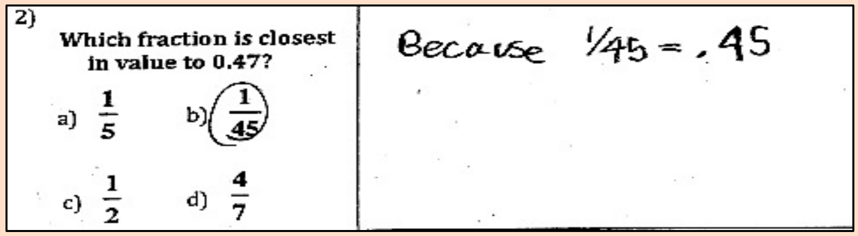
Students sometimes view a decimal number between 0 and 1 as a unit fraction with a denominator determined by the digits to the right of the decimal point.


1) Which fraction is closest in value to 0.68?

a) $\frac{1}{70}$ b) $\frac{1}{7}$ c) $\frac{2}{3}$ d) $\frac{6}{8}$

.70

A because $\frac{1}{70} = 0.70$ which is close to 0.68

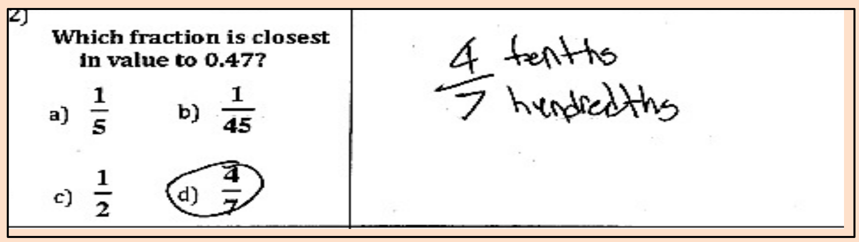

2) Which fraction is closest in value to 0.47?

a) $\frac{1}{5}$ b) $\frac{1}{45}$ c) $\frac{1}{2}$ d) $\frac{4}{7}$

Because $1/45 = .45$

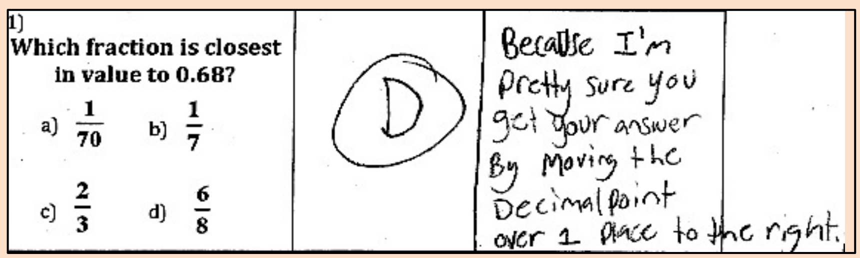
Incorrect conversion to a fraction

Students sometimes place the fraction bar between the numbers to the right of the decimal point.


2) Which fraction is closest in value to 0.47?

a) $\frac{1}{5}$ b) $\frac{1}{45}$ c) $\frac{1}{2}$ d) $\frac{4}{7}$

4 tenths
7 hundredths


1) Which fraction is closest in value to 0.68?

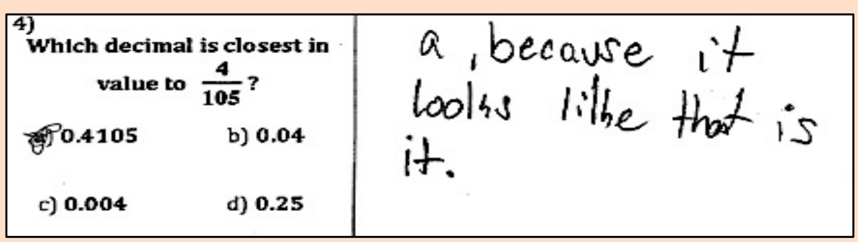
a) $\frac{1}{70}$ b) $\frac{1}{7}$ c) $\frac{2}{3}$ d) $\frac{6}{8}$

D

Because I'm pretty sure you get your answer by moving the decimal point over 1 place to the right.

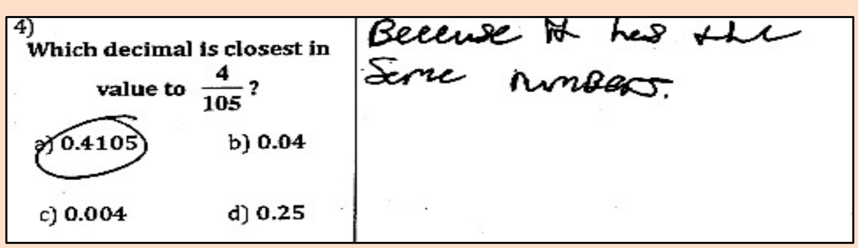
Incorrect conversion to a decimal

Students convert a fraction to a decimal as 0. followed by the numbers in the fractions.


4) Which decimal is closest in value to $\frac{4}{105}$?

a) 0.4105 b) 0.04 c) 0.004 d) 0.25

a, because it looks like that is it.


4) Which decimal is closest in value to $\frac{4}{105}$?

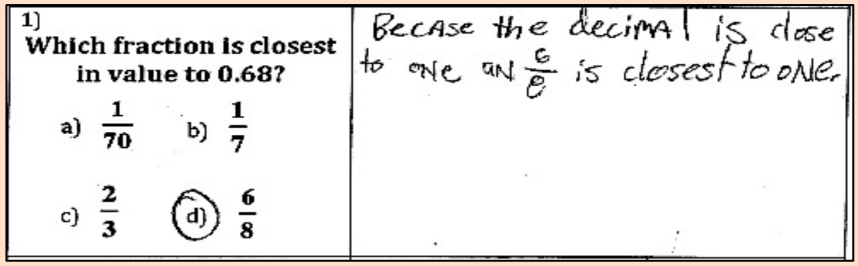
a) 0.4105 b) 0.04 c) 0.004 d) 0.25

Because it has the same numbers.

Examples that reflect common misconceptions

Imprecise or incorrect use of benchmarks

Students sometimes use reasoning about the size of the numbers in relation to a benchmark but do not consider whether other choices are closer in value.

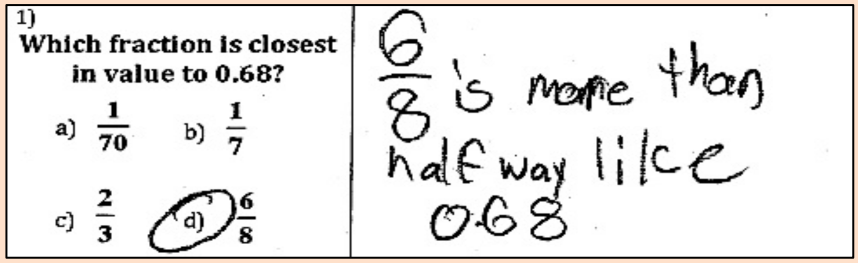


1) Which fraction is closest in value to 0.68?

a) $\frac{1}{70}$ b) $\frac{1}{7}$

c) $\frac{2}{3}$ d) $\frac{6}{8}$

Becase the decimal is close to one an $\frac{6}{8}$ is closest to one.

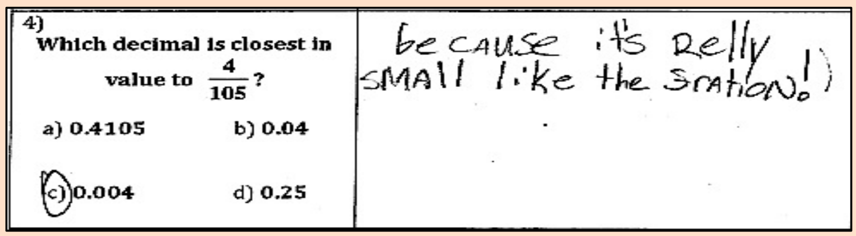


1) Which fraction is closest in value to 0.68?

a) $\frac{1}{70}$ b) $\frac{1}{7}$

c) $\frac{2}{3}$ d) $\frac{6}{8}$

$\frac{6}{8}$ is more than half way like 0.68



4) Which decimal is closest in value to $\frac{4}{105}$?

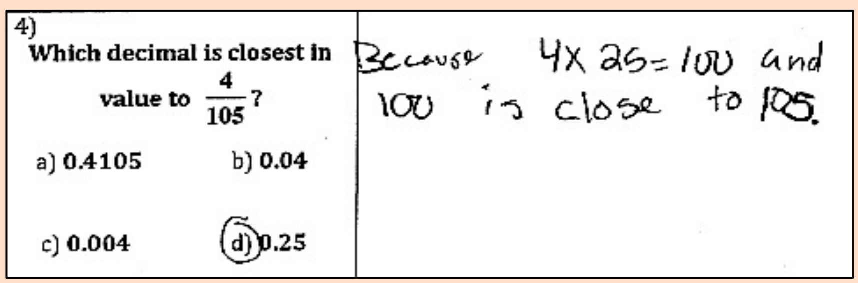
a) 0.4105 b) 0.04

c) 0.004 d) 0.25

because it's relly small like the fraction!

Denominator divided by numerator

Students sometimes look for the decimal number closest to how many times the denominator is larger than the numerator.

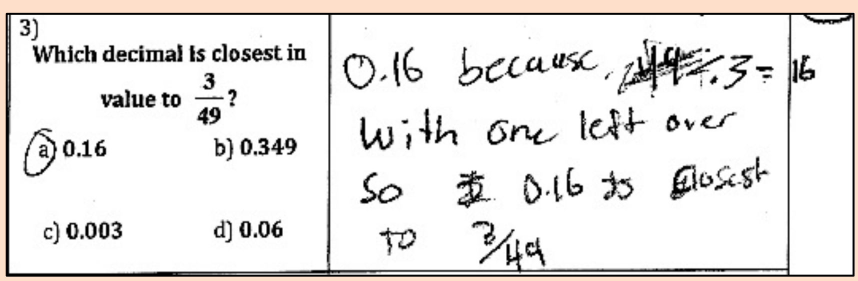


4) Which decimal is closest in value to $\frac{4}{105}$?

a) 0.4105 b) 0.04

c) 0.004 d) 0.25

Because 4x25=100 and 100 is close to 105.



3) Which decimal is closest in value to $\frac{3}{49}$?

a) 0.16 b) 0.349

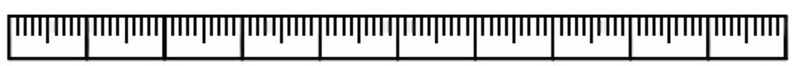
c) 0.003 d) 0.06

0.16 because, ~~49~~ ÷ 3 = 16 with one left over so 0.16 is closest to 3/49

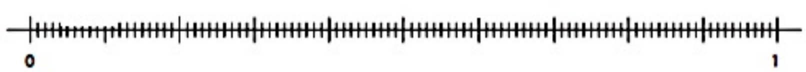
Instructional ideas to consider

- The topic of fraction and decimal equivalence includes a number of important underlying concepts and skills for students to understand:
  - A number in decimal form can be represented as an equivalent fraction because both are ways to represent the quantity.
  - Fractions with denominators that are powers of ten, such as 1/10, 27/100 or 381/1000, have a direct correspondence with decimals.
  - Fractions with denominators that are not powers of ten can still be represented as decimals. A first step is to express these fractions as equivalent fractions with denominators that are powers of ten. For example, 1/5 can be expressed as 0.2, because 1/5 is equivalent to 2/10.
  - Equivalent fractions and decimals occupy the same point on a number line.
  - Fractions and decimals can be represented using area and linear models.
- Provide opportunities for students to use place value reasoning and magnitude to compare fractions and decimals rather than always requiring students to use exact conversions.
- Anchor students' understanding of fractions, decimals and percents in concrete activities and/or contexts.
  - Use a variety of models and representations; such as base ten blocks, linear models, number lines, grid paper, area models (see examples below). Stress the underlying unifying concept of division, to help students learn the explicit links between decimals and fractions.

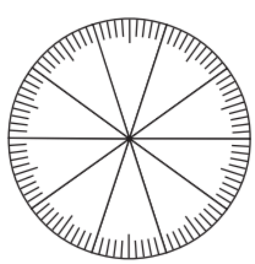
Meter Stick



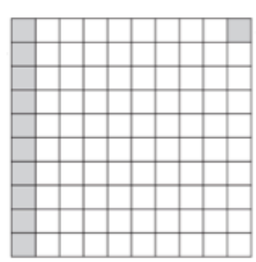
Number Line



Hundredths Wheel



Area Model



Images retrieved from:
http://eworkshop.on.ca/edu/resources/guides/NSN_vol_6_Decimal_Numbers.pdf

- Money can be used to model relationships between decimals and fractions.

- The use of interactive technology applications, such as the Gizmos interactives listed below, can facilitate the transition from concrete models to more abstract symbolic representations.

  Modeling Decimals- Area and Grid Models
  https://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=1007
  Modeling Decimals- Base Ten Blocks
  https://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=1010
  Model and compare fractions, decimals, and percents using area models
  https://www.explorelearning.com/index.cfm?ResourceID=1008&method=cResource.dspDetail

- Students with a deep and flexible understanding of fraction and decimal equivalence will chose a comparison strategy based on the specific quantities in a problem rather than applying the same strategy across all problems. Facilitate this flexibility by developing students' ability to understand the meanings and representations of part to whole relationships before teaching rules and procedures for converting between various representations.

- As always, consider which of the Mathematics Actions and Processes will be the focus of your instruction. (i.e. have students defend their choices to other students to support ability to communicate using mathematical reasoning)

Sample Hinge-point Question to Assess Progress

Here are two examples. You will likely need to create additional hinge-point questions as you implement targeted instruction to address learning needs.

1. Which decimal is closest in value to $\frac{2}{53}$?

a) 0.253 b) 2.53

c) 0.04 d) 0.25

2. Which fraction is closest in value to 0.23 ?

a) $\frac{2}{53}$ b) $\frac{1}{4}$

c) $\frac{1}{23}$ d) $\frac{1}{3}$

Correct selected response choices for Hinge-point question.

1. c
2. b