Name ________________________________

Comparing Fractions

For each problem below, circle the fraction that has the GREATER value.

Circle your choice:	Explain or show why you circled the fraction:
A) $\frac{3}{5}$ $\frac{3}{7}$	
B) $\frac{3}{4}$ $\frac{2}{5}$	
C) $\frac{2}{3}$ $\frac{7}{8}$	

Name ______________________________

Comparing Fractions

For each problem below, show how the values compare by choosing the >, < or = symbol.

Circle >, < or =	**Explain or show why you chose that symbol:**
D) $\frac{4}{7} \; \square \; \frac{4}{6}$ **Circle the symbol that goes in the □ above.** **> < =**	
E) $\frac{2}{3} \; \square \; \frac{4}{6}$ **Circle the symbol that goes in the □ above.** **> < =**	
F) $\frac{5}{6} \; \square \; \frac{7}{8}$ **Circle the symbol that goes in the □ above.** **> < =**	

Comparing Numbers: Fractions

This resource guides you in using the ACT cycle to implement this probe with your students and use the findings to plan instructional next steps.

Here is one example from this 6-item comparing numbers probe:

For each problem below, circle the fraction that has the GREATER value

Circle your choice:	Explain or show why you circled the fraction:
A) $\frac{3}{5}$ $\frac{3}{7}$	

Analyze the Assessment — What is the math?

This probe gathers information about the extent to which students use reasoning about place value and magnitude when comparing fractions.

Do Students…		
▪ Reason about the relationship between the magnitude of fractions by: ○ comparing using a common benchmark? ○ representing using an area or linear model then comparing? ○ comparing using a common denominator? ▪ Provide mathematical justifications to explain the relationship between the fractions (i.e. use of visual models, common denominators, etc.)?	**OR**	▪ Apply incorrect reasoning about the relationship between the magnitude of quantities by: ○ overgeneralizing from whole number comparisons? ○ misjudging the magnitude of the quantity as compared to a benchmark number? ○ using incorrect or imprecise representations? ▪ Provide inaccurate mathematical justifications that show lack of conceptual understanding of relationships between two fractions?

Oklahoma Academic Standards for Mathematics

Below are the associated standards related to the intended content of the probe. This may mean a direct relationship (the content directly addresses the standard), but the content focus may instead be foundational for the standard—that is, the target may be necessary before the standard can be addressed.

> **4.N.2.1** Represent and rename equivalent fractions using fraction models (e.g. parts of a set, area models, fraction strips, number lines).
>
> **4.N.2.2** Use benchmark fractions to locate additional fractions on a number line. Use models to order and compare whole numbers and fractions less than and greater than one using comparative language and symbols.

Consider Students' Thinking

Examine their work

Each probe item requires a two-part response from the student: a selected response and a written explanation using words and/or pictures. Together, these two parts provide important information about the student's understanding and thinking. Four possible combinations of student responses are described below.

- correct selected response choice AND an explanation that provides sound reasoning
- correct selected response choice AND an explanation containing flawed or no reasoning
- incorrect selected response choice AND an explanation with reasoning that reflects some understanding
- incorrect selected response choice AND an explanation containing flawed or no reasoning

In preparation for examining your own student work, review the following:

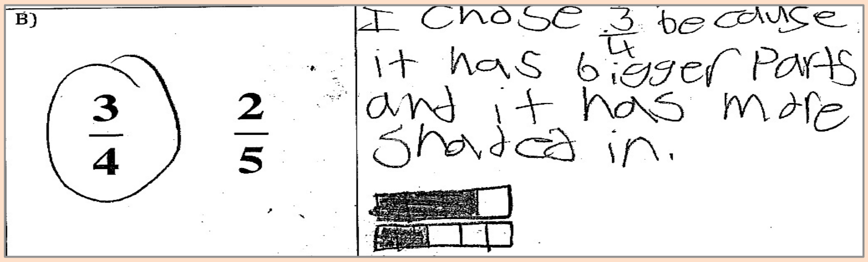
1. the correct selected response answers;
2. student work samples showing correct selected response choices supported by sound reasoning and/or successful strategies; and
3. student work samples to illustrate common misconceptions.

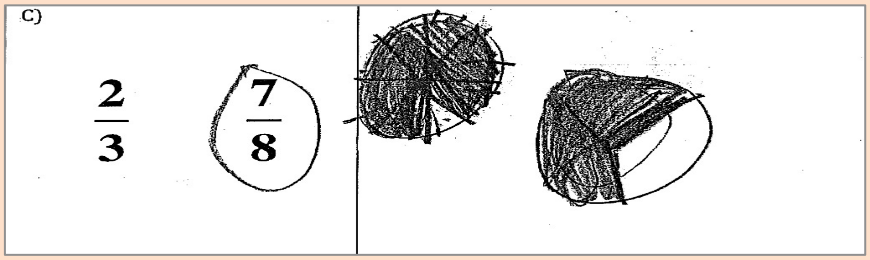
1. Correct selected response choices

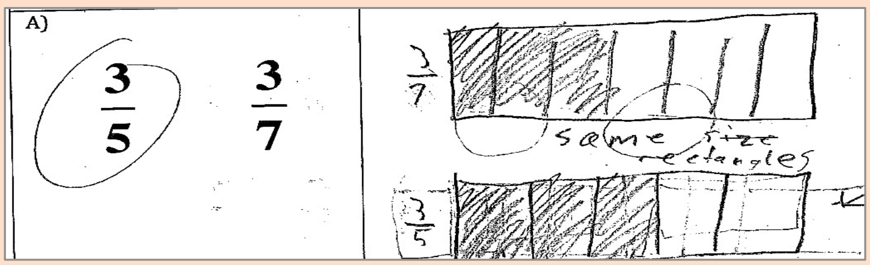
A) $\frac{3}{5}$	B) $\frac{3}{4}$	C) $\frac{7}{8}$
D) $<$	E) $=$	F) $<$

2. **Examples of correct selected response choices with sound reasoning and/or successful strategies**

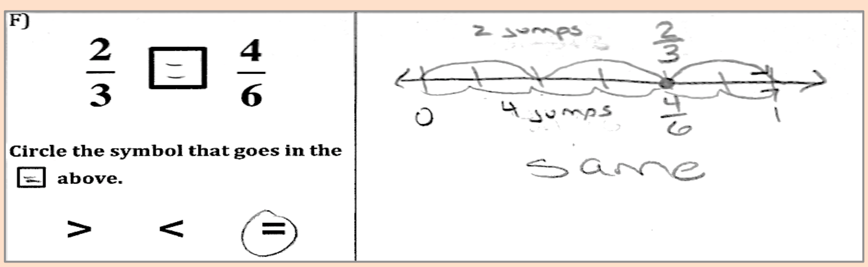
Students successfully represent and compare fractions using area models.

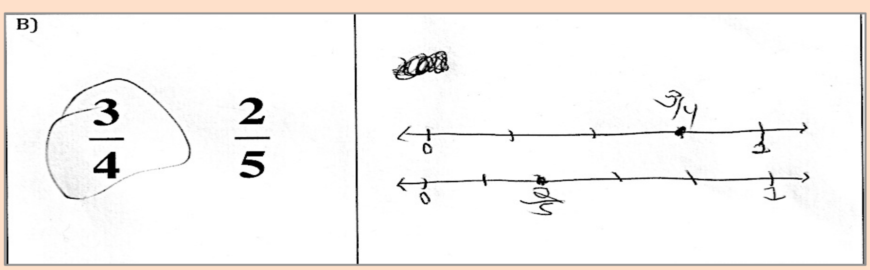






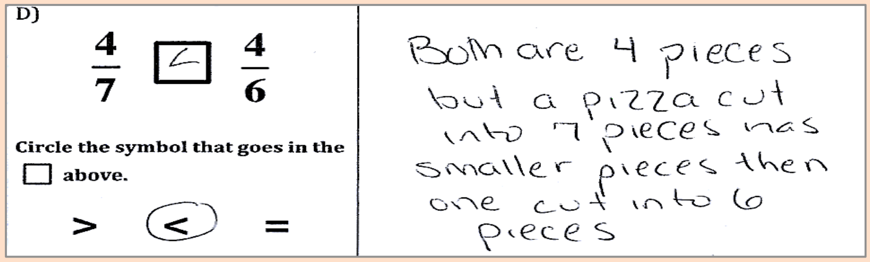
Students successfully represent and compare fractions using number lines

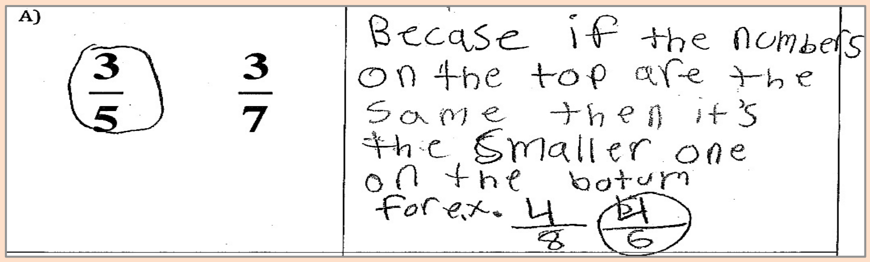




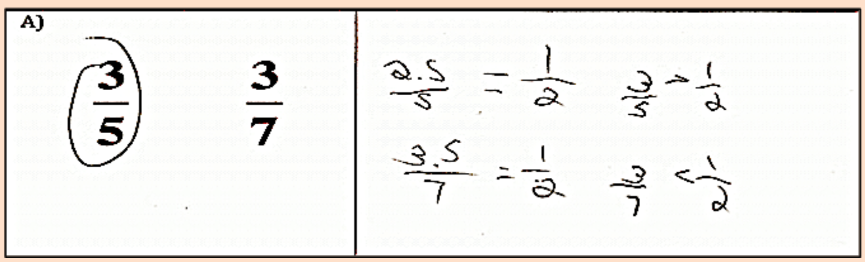
Examples of correct selected response choices with sound reasoning and/or successful strategies

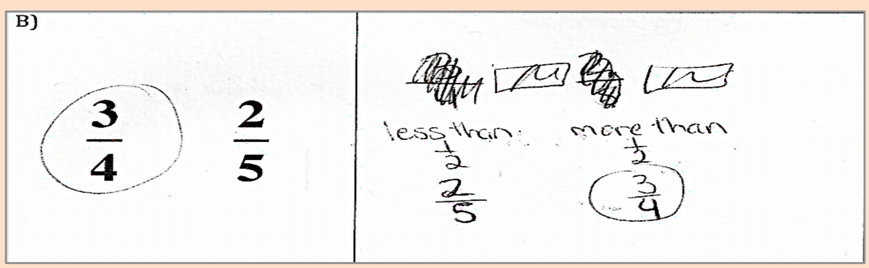
Students successfully compare fractions by reasoning about the size of the denominator given equivalent numerators.



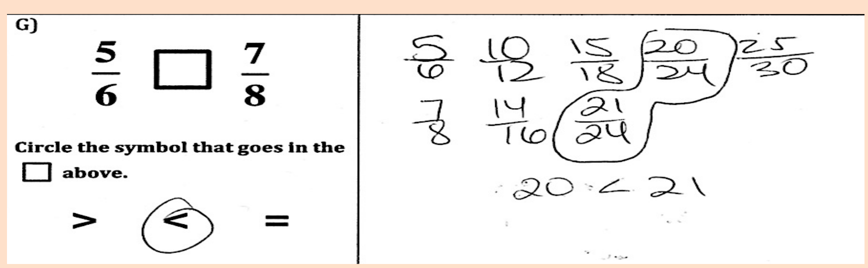


Students successfully compare fractions by each to the benchmark fraction $\frac{1}{2}$.





Student successfully compares the fractions by finding common denominators.



3. Examples that reflect common misconceptions

Overgeneralizing from whole number comparisons

Students may compare the given numerators and/or denominators and make their decision based on which has the "biggest" number.

A)

$\frac{3}{5}$ $\frac{3}{7}$

I chose $\frac{3}{7}$ because 7 is greater than 5.

F)

$\frac{2}{3}$ $\boxed{<}$ $\frac{4}{6}$

Circle the symbol that goes in the □ above.

> < =

I chose $\frac{4}{6}$ because 6 is bigger than 3.

Overgeneralizing from unit fraction comparisons

Students sometimes associate the smaller numerator with the smallest fraction (i.e. $\frac{1}{5} > \frac{1}{8}$) regardless of the number in the numerator.

B)

$\frac{3}{4}$ $\frac{2}{5}$

I chose $\frac{2}{5}$ even know it's a smaller number it will always be right.

G)

$\frac{5}{6}$ $\boxed{>}$ $\frac{7}{8}$

Circle the symbol that goes in the □ above.

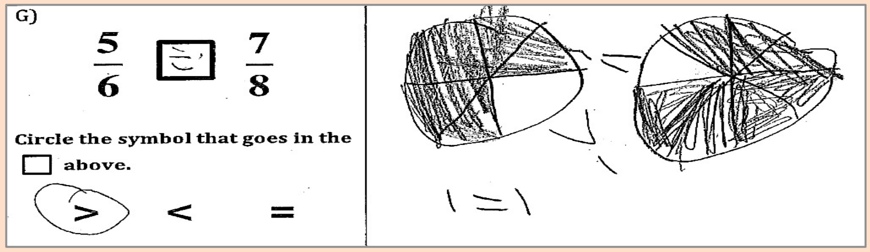
> < =

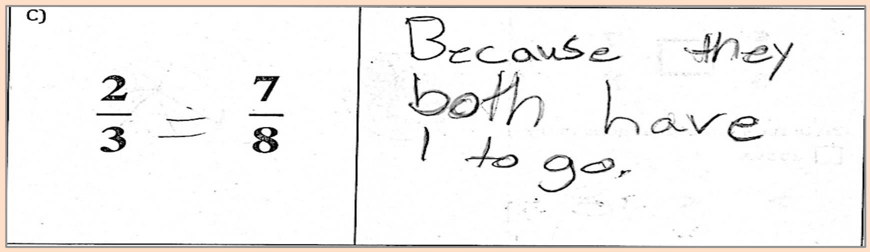
it is bigger because 5 is smaller 7 and 6 is smaer than 8 but 5 and 6 are bigger because the bigger the takeing the smaller it gets

Examples that reflect common misconceptions

Gap Reasoning

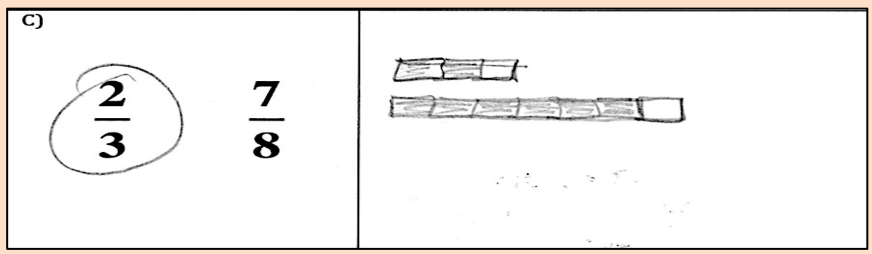
Students sometimes compare fractions by determining the number of parts needed to make a whole, rather than reasoning about the size of the parts.

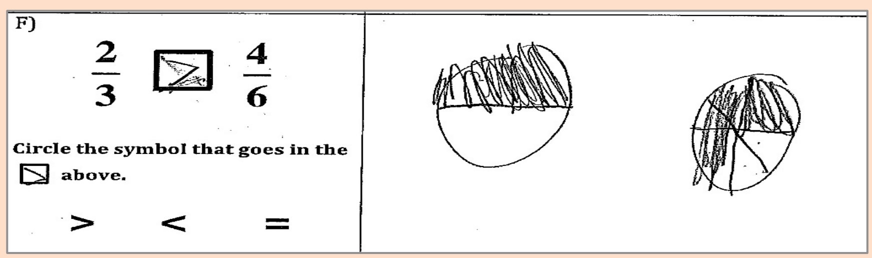


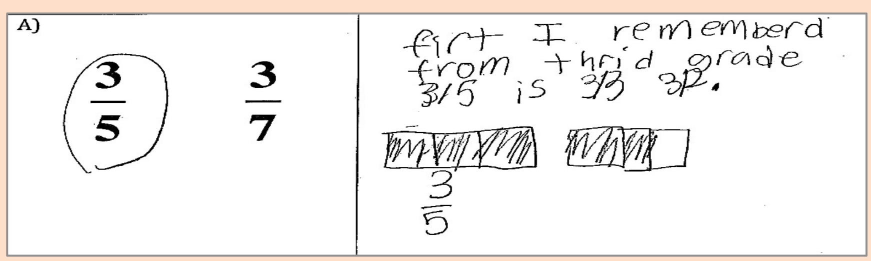


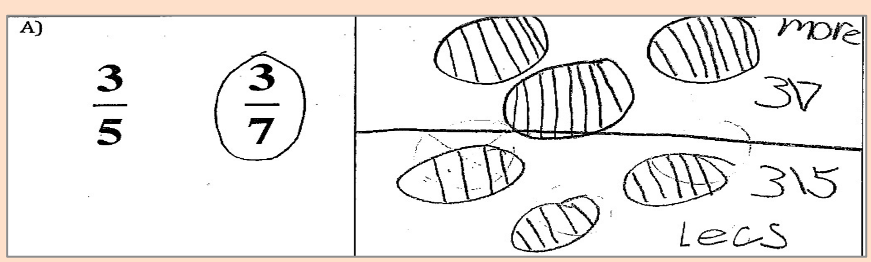
Incorrect Models

Students sometimes have difficulty representing fractions. In the first example, the student uses equivalent parts rather than equivalent wholes. In the second example, the student does not use equivalent parts when representing $\frac{2}{3}$. In third example, the student represents $\frac{3}{5}$ using two area models. In the fourth example, the student is representing the fractions as 3 wholes each spilt into n parts.





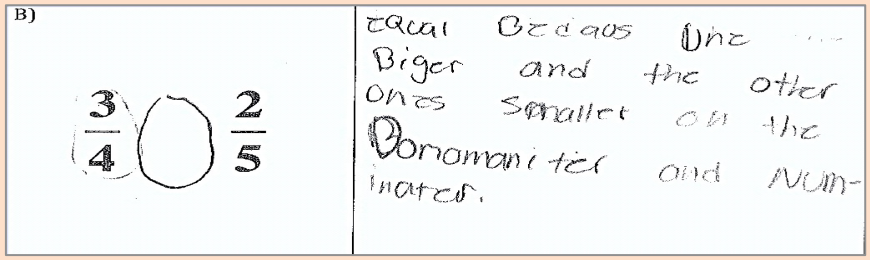


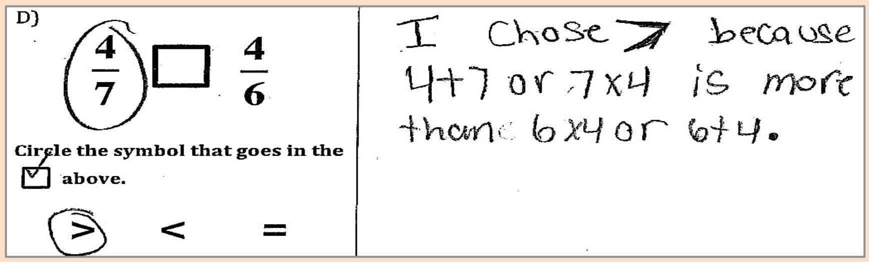


Examples that reflect common misconceptions

Additional overgeneralization approaches

The following examples demonstrate additional incorrect student strategies related to overgeneralizing from whole number comparisons. In the first example, the student applies the whole number overgeneralization to compare. Because the one fraction has a greater numerator and the other fraction has a greater denominator, the student concludes the fractions are equal. In the second example, the student compares both the product and sum of the numerators and denominators.





Take Action

Move student learning forward

Instructional ideas to consider

- The topic of fraction comparison includes a number of important underlying concepts and skills for students to understand:
  - Fractions can be represented and compared using area and linear models.
  - Comparisons can be made only when the two fractions refer to the same whole.
  - Fractions can be compared against common benchmarks such as ½.
  - Equivalent fractions occupy the same point on a number line. When two non-equivalent fractions are plotted on a horizontal number line, one of the fractions will be plotted to the left of the other fraction. The fraction plotted on the left is the smaller fraction. The fraction plotted on the right-hand side is the larger fraction.
  - When the denominators are the same, the fraction with the larger numerator is larger and if the numerators are the same, the fraction with the larger denominator is smaller
  - Fractions can be expressed in their equivalent decimal or percent form as a way to make comparisons.
  - The results of comparisons can be recorded with the symbols >, =, or <

- Students need multiple experiences comparing fractions:
  - with the s*ame denominator* to focus on the *number* of same size parts;
  - with the *same numerator* to stress the same number but *different size* pieces;
  - with unlike numerators and denominators when one fraction's denominator is a multiple of the other denominator and when the fractions have denominators are not multiples of each other;
  - that are more or less than *key benchmarks* such as ¼, or ½; and
  - that are *close to one whole*.
- Anchor students' understanding of fractions in concrete activities and/or contexts and make explicit connections between area, linear, verbal and symbolic representations.
- Students should understand why procedures such as finding common denominators and comparing by using cross-multiplication work and make sense.
- Students with a deep and flexible understanding of fractions will choose a comparison strategy based on the specific quantities in a problem rather than apply the same strategy across all problems. Facilitate this flexibility by developing students' ability to understand the meanings and representations of part to whole relationships before teaching rules and procedures for converting between various representations.
- The use of interactive technology applications, such as the Gizmos and NCTM Illumination interactives listed below, can facilitate the transition from concrete models to more abstract symbolic representations.

  Comparing Fractions- Number Lines (Gizmos)
  https://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=1004
  Compare fractions – Area Models (Gizmos)
  https://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=1006
  Equivalent Fractions: Area and Number Line (NCTM Illuminations)
  http://illuminations.nctm.org/Activity.aspx?id=3510
- As always, consider which of the Mathematics Actions and Processes will be the focus of your instruction. (i.e. have students defend their choices to other students to support ability to communicate using mathematical reasoning)

Sample Hinge-point Question to Assess Progress

Here are two examples. You will likely need to create additional hinge-point questions as you implement targeted instruction to address learning needs.

1. Which symbol can be used to compare the two fractions?

$$\frac{8}{15} \square \frac{5}{11}$$

a) < b) = c) >

2. Which symbol can be used to compare the two fractions?

$$\frac{7}{8} \square \frac{11}{12}$$

a) < b) = c) >

Correct selected response choices for Hinge-point question.

1. >
2. <